GRAVITATIONAL LENSING IN MODIFIED GRAVITY AND THE LENSING OF MERGING CLUSTERS WITHOUT DARK MATTER

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Abstract

Gravitational lensing in a modified gravity (MOG) is derived and shown to describe lensing without postulating dark matter. The recent data for merging clusters identified with the interacting cluster 1E0657-56 is shown to be consistent with a weak lensing construction based on MOG without exotic dark matter.

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1 Introduction

A relativistic modified gravity (MOG) theory [1, 2] has been proposed to explain the rotational velocity curves of galaxies and the X-ray data for clusters of galaxies with a modified Newtonian acceleration law, without non-baryonic dark matter. A fitting routine for galaxy rotation curves has been used to fit a large number of galaxy rotational velocity curve data, including low surface brightness (LSB), high surface brightness (HSB), dwarf galaxies and elliptical galaxies with both photometric data and a two-parameter core model without non-baryonic dark matter [3]. The fits to the data are remarkably good and for the photometric data only the one parameter, the mass-to-light ratio $\langle M/L \rangle$, is used for the fitting, once two parameters are universally fixed for galaxies and dwarf galaxies. A large sample of mass profile X-ray cluster data has also been fitted [4] without dark matter. It has been shown that MOG can fit the Cosmic Microwave Background acoustical oscillation peaks data in the power spectrum without dark matter and provide an explanation for the accelerated expansion of the universe [2].

The MOG requires that Newton's constant G, the coupling constant ω that measures the strength of the coupling of a skew field to matter, and the mass μ of

the skew field vary with distance and time, so that agreement with the solar system and the binary pulsar PSR 1913+16 data can be achieved, as well as fits to galaxy rotation curve data and galaxy cluster data. In MOG [1, 2], the action contains the Einstein-Hilbert action based on a symmetric pseudo-Riemannian metric, an action formed from a vector field ϕ_{μ} called the phion field which produces a "fifth" force skew field, and an action for scalar fields that leads to *effective* field equations describing the variations of G, ω and μ .

In the following, we shall investigate the gravitational lensing in MOG. The variation of G leads to a consistent description of relativistic lensing effects for galaxies without non-baryonic dark matter. We study the weak gravitational lensing of the merging, interactive cluster 1E0657-56 at a redshift z=0.296 that has recently been claimed to enable a direct detection of dark matter, without alternative gravitational theories [6, 7, 8]. We will show that the lensing of distant background galaxies predicted by MOG is consistent with the data from the interacting cluster 1E0657-56.

2 Lensing Deflection of Light Rays

In relativistic MOG massless photons move along null geodesics [1, 2]:

$$\frac{d^2x^{\mu}}{d\tau^2} + \Gamma^{\mu}_{\alpha\beta} \frac{dx^{\alpha}}{d\tau} \frac{dx^{\beta}}{d\tau} = 0, \tag{1}$$

where $\Gamma^{\mu}_{\alpha\beta}$ denotes the Christoffel symbol. The relativistic deflection of light for a point mass in MOG is given by

$$\alpha(\theta) = -\frac{\theta_E^2}{\theta},\tag{2}$$

where $\alpha(\theta)$ is the reduced bending angle, which relates the angular position of its image, θ , via the equation, $\theta_s = \theta + \alpha(\theta)$ with θ_s denoting the angular position of the source. Moreover, θ_E denotes the Einstein radius of the lens [5]:

$$\theta_E = \left(\frac{4GM}{c^2} \frac{D_{0s}}{D_{ol}D_{ls}}\right)^{1/2},\tag{3}$$

where G is the varying gravitational constant in MOG, M is the mass of the deflector, and D_{os} , D_{ol} and D_{ls} are the angular diameter distances from observer to source, observer to lens, and lens to source, respectively. In a cosmological model, the results obtained from MOG are not strongly dependent on the distance measurements.

Mortlock and Turner [9] have proposed a generic, parameterized point-mass deflection law for the weak lensing of galaxies:

$$\alpha(\theta) = -\frac{\theta_E^2}{\theta} \left(\frac{\theta_0}{\theta_0 + \theta} \right)^{\xi - 1},\tag{4}$$

which is equivalent to the general relativity (GR) Schwarzschild law for $\theta \ll \theta_0$, but falls off as $\alpha(\theta) \propto \theta^{1-\xi}$ for $\theta \gg \theta_0$. For $\xi < 0$ the deflection angle increases with impact parameter R and $\xi = 1$ for GR. For galaxy-galaxy weak lensing, Mortlock and Turner [9] found that the full Sloan Digital Sky Survey (SDSS) data constrain the deflection angle to be $\alpha(R) \propto R^{0.1 \pm 0.1}$ for 50 kpc < R < 1 Mpc. This shows that for galaxy-galaxy weak lensing the gravitational constant does not vary significantly with the impact parameter R.

The deflection angle formula can be written:

$$\alpha = \frac{2}{c^2} \int_{-\infty}^{\infty} d\ell a_{\perp}(r), \tag{5}$$

where r denotes the radial polar coordinate for a spherically symmetric body, ℓ is the distance along the ray path and $r = \sqrt{\ell^2 + R^2}$. Moreover,

$$a_{\perp}(r) = a(r)\frac{R}{r},\tag{6}$$

is the gravitational acceleration perpendicular to the direction of the photon at a distance of closest approach R from the source and

$$a(r) = \frac{d\Phi}{dr},\tag{7}$$

where Φ denotes the gravitational potential with $|\Phi| \ll c^2$. We can express the deflection angle as

$$\alpha(R) = \frac{4}{c^2} \int_R^\infty dr \frac{R}{\sqrt{r^2 - R^2}} \frac{d\Phi(r)}{dr}.$$
 (8)

The metric line element is given for weak gravitational fields by

$$d\tau^2 = \exp(2\Phi/c^2)c^2dt^2 - \exp(-2\Phi/c^2)d\ell^2,$$
(9)

where

$$d\ell^2 = dr^2 + r^2(d\theta^2 + \sin^2\theta d\phi^2).$$
 (10)

The time delay of a light ray is

$$\Delta t \approx \frac{1}{c} \int_0^{\ell_{os}} d\ell \exp(-2\Phi/c^2) \approx \frac{1}{c} \left(\int_0^{\ell_{os}} d\ell - \frac{2}{c^2} \int_0^{\ell_{os}} d\ell \Phi \right), \tag{11}$$

where ℓ_{0s} denotes the distance from observer to source. The deflection angle θ obtained from Eq.(8) is twice the deflection angle experienced by a massive particle moving with the speed of light [10].

In MOG the acceleration on a test particle is given by [1]:

$$a(r) \equiv \frac{d\Phi(r)}{dr} = -\frac{G(r)M(r)}{r^2},\tag{12}$$

where G(r) is the *effective* varying gravitational "constant":

$$G(r) = G_N \left[1 + \alpha(r) \left(1 - \exp(-r/\lambda(r)) \right) \left(1 + \frac{r}{\lambda(r)} \right) \right]. \tag{13}$$

Here, G_N denotes Newton's gravitational constant and $\alpha(r)$ and $\lambda(r)$ denote, respectively, the "running" coupling strength and range of the vector "phion" field in MOG. We have for $r \to \infty$ that $G_\infty \to G_N(1+\alpha)$, while $G(r) \to G_N$ as $r \to 0$.

The geometry of a spherical lens at a redshift $z=z_l$ bends the light ray from a source at redshift $z=z_s$. The source is offset from the lens by an angle θ_s and forms an image at an angle θ , which is related to the length R by $\theta=R/D_{ol}$. The spherical symmetry of the lens means that the line of sight to the lens, source and image lie in the same plane. The lens equation is

$$\theta - \theta_s = \frac{D_{ls}}{D_{os}}\alpha. \tag{14}$$

The convergence κ and shear γ are defined by

$$\kappa = \frac{1}{2\theta} \frac{\partial}{\partial \theta} (\theta^2 \bar{\kappa}), \quad \gamma = |\kappa - \bar{\kappa}|, \tag{15}$$

where

$$\bar{\kappa} = \frac{2}{\theta^2} \int_0^\theta d\theta (\theta \kappa) \tag{16}$$

is the mean convergence within a circular radius.

3 Cluster Lensing and the Lensing of Merging Clusters

Weak gravitational lensing is a method that can be employed to measure the surface mass in a region by using the fact that a light ray passing a gravitational potential will be bent by the potential. Images of background galaxies that are near a massive cluster of galaxies are deflected away from the cluster and enlarged while preserving the surface brightness. The images are distorted tangentially to the center of the gravitational potential and produce a shear γ , causing the background galaxies' ellipticities to deviate from an isotropic distribution; the magnitude and direction of these deviations can be used to measure the mass of the cluster causing the lensing. No assumptions need be made about the dynamical state of the cluster mass.

The measured shear can be converted into a measurement of the convergence κ , which is related to the surface density of the lens Σ by the equation:

$$\kappa(R) = \frac{\Sigma(R)}{\Sigma_{\text{crit}}(R)},\tag{17}$$

where $\Sigma_{\text{crit}}(R)$ is a scaling factor in MOG:

$$\Sigma_{\rm crit}(R) = \frac{c^2}{4\pi G(R)D}.$$
 (18)

Here, $D^{-1} = D_{0s}/(D_{ol}D_{ls})$ and we have explicitly included the variation of G with the impact parameter R.

A weak lensing reconstruction of the interacting cluster 1E0657-56 shows that a smaller cluster has undergone in-fall and passed through a primary cluster. The interacting cluster has previously been identified in optical and X-ray surveys [6, 7, 8] using the Chandra X-ray Observatory. During the merger of the two clusters of galaxies, the X-ray gas has been separated from the galaxies by ram-pressure and is observed to be off-set from the center of the interacting cluster. The merger is occurring approximately in the plane of the sky and the cluster cores passed though each other $\sim 100\,\mathrm{Myr}$ ago.

If dark matter exists in clusters of galaxies, then during the collision of two clusters the hot X-ray emitting gas of the clusters made of baryons is slowed by a drag force, whereas the collisionless dark matter and the galaxies made of ordinary matter will not be slowed down by the impact of the clusters, producing the observed separation of the dark matter and normal matter in galaxies from the normal matter associated with the X-ray gas. The mass of the X-ray emitting gas is at least 7 times larger than the ordinary matter of the galaxies, so that in Einstein's and Newton's gravity theories additional dominant dark matter is required to fit the interacting cluster data. However, as we shall see in the following, MOG predicts a length dependent scaling of gravity such that the gravitational field at the positions of the ordinary galaxy matter is increased in strength, predicting the peaking of the weak lensing without dark matter.

We shall simplify our analysis of the merging clusters by assuming that the observed interacting cluster is approximately spherically symmetric. The total surface density of the interacting cluster is given by

$$\Sigma(R) = \Sigma_X(R) + \Sigma_G(R), \tag{19}$$

where Σ_X and Σ_G denote the X-ray emitting gas and galaxy surface densities, respectively. The varying G(R) is given by

$$G(R) = G_N \left[1 + \alpha_{\text{clust}} (1 - \exp(-R/\lambda_{\text{clust}})) \left(1 + \frac{R}{\lambda_{\text{clust}}} \right) \right], \tag{20}$$

where α_{clust} and λ_{clust} are the coupling strength and range of the interacting cluster, respectively, and we assume that they are constant within the interacting cluster. The convergence field κ is given by

$$\kappa(R) = \frac{4\pi G_N D}{c^2} \left[1 + \alpha_{\text{clust}} (1 - \exp(-R/\lambda_{\text{clust}})) \left(1 + \frac{R}{\lambda_{\text{clust}}} \right) \right] \Sigma(R). \tag{21}$$

We choose the phenomenological models for Σ_X and Σ_G :

$$\Sigma_X(R) = A \exp(-BR), \quad \Sigma_G = C,$$
 (22)

where A, B and C are constants. The model for the surface density Σ of the interacting cluster has been chosen to reflect the data, showing that the X-ray gas attenuates as R increases towards the edge of the merging clusters, and we have chosen a constant value for the surface density of galaxies Σ_G . We are required in a realistic model to cut off the surface density Σ at the edges of the interacting cluster. In Fig 1., we display a calculation of $\kappa(R)$ with $\alpha_{\rm clust}=13$, $\lambda_{\rm clust}=200\,{\rm kpc}$, A=B=0.6, C=0.1 in appropriate units and the numerical factor $4\pi GD/c^2$ is scaled to unity in appropriate units. The choices of the parameters $\alpha_{\rm clust}$ and $\lambda_{\rm clust}$ agree approximately with the previously published values of these parameter used to fit mass profiles of X-ray clusters [4].

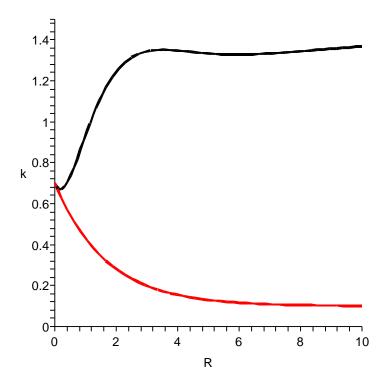


Figure 1.

Shown is a calculation of the convergence field $\kappa(R)$ for a spherically symmetric model of the interacting cluster. The MOG result is displayed by a black curve and the Einstein (Newtonian) result by a red curve. The vertical axis is displayed with the constant numerical factor $4\pi G_N D/c^2$ scaled to unity in appropriate units and the horizontal axis $R = 100 \times \text{kpc}$.

We see that the MOG prediction for the gravitational convergence field $\kappa(R)$ displays the peaking of the weak lensing in the outer regions of the interacting cluster relative to the peaking of the central off-set X-ray gas without dark matter. This agrees qualitatively with the mass distribution peaks observed in the data for the interacting cluster 1E0657-56 [6, 7, 8]. The predicted $\kappa(R)$ based on Einstein (Newtonian) lensing without dark matter cannot fit the observed distribution of surface density. This means that MOG can describe the merging clusters without assuming the existence of undetected exotic dark matter.

4 Conclusions

The gravitational potential determined from the STVG MOG [1] has a unique signature for a merging of galaxy clusters. Other alternative models based on Milgrom's MOND [12] and Bekenstein's and Sander's relativistic generalizations of MOND [13, 14, 15] have been studied by several authors [9, 10, 11]. In a MOND-type model and in the Bekenstein and Sanders models, the MOND critical acceleration a_0 is expected to satisfy $a \gg a_0 \sim 1.2 \times 10^{-8} \, \mathrm{cm \, s^{-2}}$ inside the interacting cluster, while the MOND modified acceleration law comes into play outside the interacting cluster for $a \ll a_0$. It therefore seems difficult to understand how the lack of peaking of the off-set central X-ray gas cloud compared to the more pronounced peaking of the outer galaxy mass distribution can be explained by a MOND-like model. In view of this, MOND-like models would be expected to predict an Einstein (Newtonian) gravitational field for the weak lensing of the interacting cluster, requiring dark matter to fit the data. It is already known that MOND does not fit the mass profiles of X-ray clusters without dark matter [14, 15], whereas MOG has been shown to fit a large number of mass profiles of X-ray clusters without dark matter [4]. With the accumulation of more data for 1E0657-56, this interacting cluster could distinguish MOG from other alternative gravity theories which purport to fit galaxy and clusters of galaxies data without dark matter. A more detailed study of the MOG prediction for the interacting cluster based on a fitting to published data will be considered in a future publication.

We learn from the results presented here that one should not draw premature conclusions about the existence of dark matter without a careful analysis of alternative gravity theories and their predictions for galaxy lensing and cluster lensing, in particular, for the interacting cluster 1E0657-56.

Acknowledgments

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